Title: Generative topics: building a curriculum around big ideas.
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Data from the Third International Mathematics and Science Study Mathematics and Science Study (TIMSS) point out "a comparative lack of focus and coherence in the American mathematics curriculum" and an absence of "meaningful connections between the big ideas of mathematics" (Schmidt 1997). These findings, along with those reported in other international comparisons (e.g., Stevenson and Stigler 1992), suggest that we must carefully examine the actual content of both our written and our implemented mathematics curriculum to be sure that it gives students the focus and coherence they need to develop mathematical understanding. Although teaching methodology is important, no issue is as fundamental to the heart of teaching or as important to reform in mathematics education as our decision about what to teach.

What we teach is more than a list of topics - addition, subtraction, measurement, place value, and so on. What we teach encompasses the flavor, focus, and spin that we place on these topics. For example, teaching place value as a scheme for organizing and recording quantities is fundamentally different "content" from teaching place value as a set of ordered names to be matched appropriately to a series of columns. Consequently, two teachers working from the same syllabus can teach radically different content depending on the emphasis, context, and applications that they give each topic. This subtle difference between "topics" and actual "content" makes the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) challenging to implement. Merely teaching the topics mentioned in the Standards document does not necessarily translate into achieving the vision of the Standards or providing a coherent curriculum.

For systemic reform efforts to be effective, this question of what to teach must be answered in a meaningful way by many people - administrators, curriculum specialists, parents, policy makers, and teachers. However, individual teachers charged with implementing the curriculum have an immediate need and opportunity to wrestle with this question. Although curriculum specialists and research groups offer teachers guidance and well-developed resources, effective teaching requires more than transmitting the plans, ideas, or knowledge of others. Even the best-laid-out curriculum or a well-planned unit may fail if teachers do not understand the essence of what they are teaching, bring passion to the topic, and make the content meaningful. Teachers play an important decisionmaking role in the actual implementation of curriculum, making the connections that can turn a fragmented curriculum of skills, facts, and knowledge into a coherent quest for understanding.

So how can teachers focus their curriculum and identify powerful mathematical content?

This article explores one productive avenue by which teachers, as well as principals and supervisors, may grapple both collectively and individually with the question of what to teach. Through examples of teachers' exploring the generative nature of traditional curriculum topics, this article shows how teachers can come to a better understanding of why an individual topic is important, how that topic helps build mathematical power, and what opportunities exist for making connections and supporting transfer. A framework for discussing curriculum and refraining a traditional mathematics unit is given, as well as a discussion of how this refocusing can change instruction.

Where to Start?

The NCTM's Standards document is an excellent starting place, focusing on big ideas at the core of mathematics: problem solving, communication, reasoning, and connections. These "habits of the mathematician" suggest both a methodology for our teaching and a framework for students' learning of mathematics, and act as the glue that holds the curriculum together. Although the Standards offer a loose curricular framework by specifying important broad mathematical strands and diverse topics, individual teachers must personalize and develop their own sense of curricular ownership to be effective. We must seek clarity for ourselves before we can impart it to our students.

Imagine that you have just been hired to teach second grade at a new school. At the opening faculty meeting, your principal informs you that the money for textbooks has been set aside to buy resources and materials for teaching. No standardized tests are planned. Instead, the school will decide how best to assess students' understanding and convey this information to parents. Later that afternoon, you sit down with your new colleagues to plan the mathematics curriculum. How do you decide what is important to teach?

Such a scenario may either excite or terrify you. Although it may be removed from your present situation, every year more and more schools engage in just such an enterprise on one level or another. Unfortunately, in the face of such enormous opportunity, many teachers either look back to what they had done previously or look to what others are doing, rather than look forward to what can give students real power.

The following conversation took place among a group of urban teachers from Denver, Colorado, involved in redesigning a mathematics unit on measurement for their combination first- and second-grade classrooms. The group begins the search for a generative topic by discussing what is involved in the process of measuring. The teachers reflect on how measurement is used and what it looks like in its simplest forms. For the moment, they leave behind their notions of what they had done in past measurement units, their favorite lessons, and even the district's objectives. They will address those elements later. Their first goal is to open up their perceptions of the topic.

Carl: Well, first you have to decide what to measure, don't you?

Joan: No, I don't think so. You have to have a reason for measuring first. You want to find something out, and you have to use measurement. It has a lot to do with the context, with the situation.

Lee: It seems like when you measure, you're often comparing something, like to see if it fits or if you have enough in comparison to what you need. That provides both your reason and your decision about what to measure.

Carl: There are lots of things that could be measured. That's important to think about. You would need to be aware of these different things. You could measure length, area, volume, weight, ...

Juanita: This goes back to having a reason for measuring and then deciding which of these things or characteristics to measure. Area, length, and volume are like attributes you choose from.

Lee: But a lot of the time you don't even have to measure, I don't, and I don't think kids do either. If you want to find out if someone's taller or if something fits, you just compare them. There's often no reason to go any further than comparing.

Joan: You're right, but that comparing is still measuring. You're finding out something about the two objects. If you're comparing to see who is taller, then you're dealing with height.

Carl: But is it measurement if you don't have a number to go with it?

Juanita: Sure it is. I can tell that my coffee mug holds more than your mug by measuring visually. I think you're measuring whenever you're making a comparison.

Lee: Maybe that's our generative topic, comparing. It seems to capture what we do and what kids do naturally when you measure. It's a lot more informal and natural than how I've traditionally approached measurement. My big question now is, Where do the numbers come in to all of this?

Carl: Well, there are times when you need to measure and get some kind of number to quantify that measurement. Like if you're buying wallpaper, you measure your wall and then buy the paper. If you had the paper already, you might not need to measure.

Joan: That's if you had a lot of paper; with my luck I'd run out halfway through. In your example of wallpaper, you can't directly compare things, so you have to measure and get a number.

Lee: That's the idea of transitivity that we read about. Remember? It was in that book of research ideas. If "a" is longer than "b" and "b" is longer than "c," then we know "a" is longer than "c." But we have to remember that the concept of transitivity is developmental. All of our kids might not be there yet.

Juanita: But I think we can still talk about it, since it really points up the need for measurement by comparing things you can't just match up or see automatically. I think kids can think about the idea of comparing right away. It's something that they do naturally. All we have to do is help focus their attention on the different attributes or characteristics that can be measured.

Joan: It's a new way for me to think and teach about measurement, but you're right, I think kids will find it more accessible. I also like all of the connections we've talked about already: attributes, transitivity, numbers. . . .

Through their discussion, the teachers come to a better understanding of the processes and contexts of measurement. In the process, they find a more generative way in which to frame the unit - as a unit about making comparisons. Perrone (1994) describes a generative topic as "those ideas, themes, and issues that provide the depth and variety of perspectives that help students develop significant understandings." Perkins (1992) puts forth three essential criteria by which the generativity of a topic can be judged: centrality to the discipline, richness of connections, and accessibility to the students. A generative topic, therefore, is the origin from which investigations are spawned, new concepts and understandings are produced, and connections are created. A generative topic is the parent idea pregnant with possibilities for explorations.

The idea of a generative topic as a driving force of curriculum has a long tradition in education. Dewey (1964) and Whitehead (1929) advocated teaching essentials that were relevant to children and captured their natural curiosity. The Coalition of Essential Schools finds generativity through posing essential questions (Cushman 1989). For our group of teachers, essential questions for their unit might be "How can we compare objects?" or "What does it mean to talk about less and more?" Finally, the National Science Foundation-funded project Teaching to the Big Ideas has been involved in working with elementary teachers to understand and teach the "big ideas" of mathematics (Edwards 1994).

Why a Generative Topic, Not a Theme?

Teachers have long taught mathematics and other subjects through themes. Although a theme can make a topic more accessible and interesting to students, it does not meet the criteria of a generative topic because themes lack centrality to the discipline. For example, a teacher may decide to use the theme of whales. The unit might involve measuring whales, locating whales on a map with a coordinate grid, collecting data on whales, and so on. A lot of mathematics is present here, but not a central mathematical idea. Themes are vehicles for applying skills, and perhaps even learning new skills, but they pull away from the discipline of mathematics itself. Mathematics is used as a tool to learn about something else. Such work can be a valuable and an engaging learning experience for students as long as teachers keep a tight rein on the theme and do not let it overwhelm the disciplinary content.

In comparison, a generative topic is concerned first and foremost with the core ideas of the discipline, building disciplinary understanding by focusing on these central ideas. Figure 1 presents a series of questions to help a teacher better understand the generative nature of any standard curricular topic, such as probability, estimation, fractions, and so on. A generative topic should capture the essence and spirit of what the subject or topic is all about, why it is important, and how it is used. Identifying the central core of a mathematics topic requires looking beyond the basic skill level of traditional mathematics instruction to explore how it is used in real life. Through a generative form of the topic, students and teachers should feel invited to actively engage in and apply the topic. It should make the topic accessible by connecting to students' knowledge of the word. By being accessible, a generative topic is also motivating. Students can connect the topic to their experience and engage in its application immediately. A generative topic - whether expressed as a question. a word, or a phrase - should encourage understanding, extensions, and connections.

Teaching with a Generative Topic

On the surface, the creation of a generative topic appears to be only a cosmetic change. After all, what does it matter what we call it if it is still the same unit? The power of generative topics lies in their ability to help teachers reconceptualize what they are teaching and view it from a fresh perspective. The generative topic helps identify both the real-life connections and the connections with other mathematical topics. Teachers must then act on these connections if the use of generative topics is to make a difference in students' formation of connections and transference of knowledge and skills.

By focusing students' attention on making comparisons from the start, the teachers in our example move beyond an emphasis on the acquisition of discrete measurement skills to a richer, more dynamic conception of measurement. Rather than teach students measurement techniques for determining length or volume, the teachers want to lead their students to recognize that an object may have many attributes that can be measured or compared. The teachers know that they will need to help their students acquire the language to talk about and identify these attributes.

To help students focus on measurement attributes and develop measurement vocabulary, the teachers began the unit with a series of whole-class lessons that had students explore how two objects are the same and how they are different, for example, a pencil and a comb. As students listed the many attributes, they recorded them on the chalkboard to help build vocabulary. During this initial activity, length was mentioned as an attribute, but not width. This activity was repeated with several other objects to develop students' observation skills.

The next day, the comparison question was modified. Instead of looking at "same and different," students were asked, "What does this object have more of than does this one?" Students were inventive in their answers as they compared the comb and the pencil - more color, more teeth, more points. Finally, someone said, "It is more tall." The teacher seized this opportunity to discuss the observation with the students: "Yes, this one is taller. We can say that the comb has more height than the pencil. What about looking across? If up and down is height, we sometimes call across width. Does the comb have more width than the pencil?" As students explored the more-less game with various objects, they gradually became adept at noticing attributes having to do with measurement - height, width, weight, area, and volume. Students came to see that most objects can be measured on multiple fronts.

The notion of comparing also helped the teachers remain focused on the meaningful application of measurement as a means of comparison. The teachers planned to put students in circumstances in which they must determine rather than be told what attribute to measure. To move beyond a direct comparison of two objects, the teachers wanted to design situations in which objects could not be directly compared. Students needed to develop some sort of quantitative measure. To motivate students, the teacher played the more-less game with two immovable objects, a mark on the chalkboard and a poster taped to the wall. Initially, students estimated to say that one had more length than the other, but disputes arose in the room. The teacher challenged the students to determine the answer definitively. As if by magic, students spontaneously began to measure by using their forearms, pencils, a sheet of paper, or a train of Unifix cubes. The dispute was resolved, and the realistic context of measurement was established.

Throughout the unit, students engaged in what Perkins (1992) calls "understanding performances" in which they actively applied, constructed, generalized, and contextualized their understanding as opposed to practicing rote skills. Since the teachers view learning as a product of thinking, they reviewed activities and lessons, asking themselves, "Will students need to think to do this lesson, or will they merely be repeating and mimicking my actions without understanding?" Some favorite and seemingly successful activities were rejected or modified to stimulate more thinking. For example, the teachers decided that it was not appropriate to ask children to measure the length of their desks without any reason, so they abandoned the worksheet that guided students through such an activity.

In watching students work, the teachers clearly saw that they were very comfortable with the idea of direct comparison and were becoming more familiar with the new language of measurement. The teacher posed a new question for students to explore: "You said that this object has more length. How much more?" A series of rich discussions ensued in which students moved from relying on informal notions to devising a plan for comparing the objects by using beans, pennies, and Unifix cubes. Teachers modified this question to encourage students to explore a variety of measurement attributes.

Assessing a Generative Topic

In the past, the measurement units that the teachers in Denver had taught did not have a strong assessment component. The work usually progressed from one measuring-and-recording activity to another, with the culmination being yet one more similar activity. Because the teachers were not clear about what they wanted their students to understand about measurement and did not know how their activities built toward such an understanding, both ongoing and summative assessment suffered. The focus was on measuring accurately rather than on understanding the concept and the multifaceted nature of measurement. Although the previous assessments told what students could do - measure correctly - they did not tell whether students recognized when to measure and what to measure.

Using their new understanding of measurement to direct their efforts, the teachers decide that at the end of the unit, students should be able to pick any two objects and tell multiple ways in which those two objects might be compared. Some of those attributes should be attributes of measurement. Students should then be able to identify "how much more" one object had on any number of measurement-related attributes. To prepare students for this activity, the teachers introduced new measurement activities involving measuring area with tiles and measuring volume with cubes.

In their final assessment, students selected two objects and compared them in as many ways as possible. They used their data to determine which was bigger - a task that they now understood requires a judgment. Finally, they justified their selection with a written response. Having been actively immersed in the process of making such comparisons and judgments, the students were able to participate in an assessment that mirrored the instruction and fostered a high probability of success. The element of choice gave students an opportunity to show their imagination and to exercise ownership over the problem. In evaluating the work, the teachers made their own estimates and looked closely at students' recording. It was evident who had a rich conception of measurement and who was still comfortable exploring only length and width.

Making Multiplication and Division More Generative

A group of third- and fourth-grade teachers discussed the topic of multiplication. Quickly they came to the consensus that multiplication is really a means of counting things in groups. This notion opened up the conversation, and they were soon talking about division. In thinking about grouping situations, the teachers realized that many problems can be solved by either division or multiplication, depending on how a person thinks about the situation.

They noted that their district curriculum had divided these topics but that they could be reunited through the generative topic of "grouping." In thinking about multiplication and division in this way, they recognized that the concepts are accessible to all their students. Grouping relates to students' lives in many ways and offers rich opportunities for exploration. By moving away from their traditional skills-oriented approach and toward a problem-solving emphasis, the teachers were more comfortable teaching the unit to their multiage classes. As planning progressed, the teachers sought ways to put students in situations that would require them to group objects - such as sharing a box of crackers among friends or counting out a large quantity.

Initial experiences involved the class members in finding different ways to line up in rows of equal length. As students explored this problem with manipulatives, the teachers observed them dealing with the concepts of factors, primes, remainders, addition, and subtraction. From the start, students made rich and meaningful connections while developing confidence in problem solving. Many such experiences were explored, and soon the term and the symbol for multiplication were introduced as shorthand to be used instead of writing out "rows of," "groups of," or "stacks of." With a better understanding of multiplication and division, the teachers were able to devise a unit that involved students in challenging problems from the beginning - a dramatic departure from the way that they had previously taught multiplication.

Conclusions

Generative topics are one avenue for opening up the curriculum, creating connections, and providing means for transfer. They also help refocus and reorient the mathematics curriculum in a way that is consistent with current calls for systemic reform (Ewing and Sharp 1996; Stevenson and Stigler 1992). In addition, generative topics respond to Whitehead's (1929) call of seventy years ago to "let the main ideas which are introduced into a child's education be few and important." However, it is important to remember that much of the power of generative topics lies in the dialogue surrounding their formation. By discussing the mathematics they teach and seeking to discover why it is important, how it is used, and its basic essence, teachers can move beyond teaching discrete and often isolated skills. Simply renaming a unit is not enough to change its focus. The generative topic should invite a more open-ended exploration of the topic that emphasizes new goals in terms of understanding and application.

A move toward the use of more generative topics in mathematics helps us realize the vision of the Standards document (1989) by placing greater emphasis on mathematics as connections, problem solving, reasoning, and communication. The connections are evident in the topic itself and are an integral part of instruction. Students are invited to learn through problem solving as they explore and rediscover the topic in a natural context. As this exploration expands and students engage in understanding performances, a sense of inquiry is developed in which both questioning and reasoning about the topic are encouraged and enhanced. Throughout, students are encouraged to build on their everyday language as they gradually integrate the appropriate mathematics vocabulary into their discussions.

Through the regular use of generative topics, students develop a sense of curiosity about, and appreciation of, mathematics while acquiring skills within the context of their application rather than in isolation. Through generative topics, both students and teachers can explore and exploit a world of deep, rich, and powerful mathematics that has both focus and coherence.

FIGURE 1

Questions to consider in creating a generative topic

Reframing a Traditional Unit

1. Centrality: Why is this topic important?

* What does it allow us to do?

* Where can it take us?

* Humankind invented _____ to solve what problem or to facilitate what activities?

2. Accessibility: What is this topic really all about?

* How would you describe this topic in everyday language as opposed to mathematics language?

* How would students invent or discover this topic on their own? In what context?

* What do you do when you engage _____?

* How does this topic appear in real life?

3. Richness: How is this topic connected to other topics in and outside of mathematics?

* What skills and knowledge must you draw on as you apply and use this topic?

* In what other areas is this topic used?

* Understanding of this, topic opens up what new areas?

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